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ELEMENTARY PARTICLES AND FIELDS
Experiment

Searches for a Sterile-Neutrino Admixture in Detecting Tritium Decays in a Proportional Counter: New Possibilities

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Abstract—An experiment aimed at searches for an admixture of a sterile neutrino whose mass is 1 to 8 keV via detecting electrons from tritium decay in a proportional counter is proposed. The admixture in question can be discovered by a specific distortion of the energy spectrum of these electrons. For the above masses, the distortion extends over the whole spectrum; therefore, use can be made of detectors that have a relatively low energy resolution (about 10 to 15%). A classic proportional counter is a simple and natural choice of detector for the decays of a gaseous tritium. The approach that we propose is novel in two respects. On one hand, the proportional counter used is made as a discrete unit in the form of a fully fused quartz tube. This permits a readout of current signals directly from the anode filament and ensures a high stability in the case of long-term measurements. At the same time, the application of state-of-the-art digital data-acquisition methods will make it possible to perform measurements under conditions of high counting rates—up to 10^6 Hz. As a result, the energy spectrum of electrons from tritium decays that is formed by 10^{12} counts could be accumulated within about a month. This data sample would make it possible to set an upper limit in the range of 10^{-3}–10^{-5} on a sterile-neutrino admixture at a confidence level of three standard deviations (3σ) for m_s in the range of 1–8 keV, this being one to two orders of magnitude more stringent than present-day limits.

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1. INTRODUCTION

A nonzero neutrino mass and the existence of dark matter, whose nature is not known presently, indicate that the Standard Model of particle physics is incomplete. Sterile neutrinos are among the most natural candidates for dark-matter particles. They appear in the majority of extensions of the Standard Model that include massive neutrinos. Sterile (right-handed) neutrinos are neutral leptons. They are $SU(2)$ singlets and therefore do not take part in weak interactions. With the exception of some special degenerate cases, however, sterile neutrinos always undergo mixing with active (left-handed) neutrinos, thereby leading to the appearance of new mass states in the spectrum. From the experimental point of view, the results of some short-baseline experiments aimed at searches for neutrino oscillations (LSND and MiniBooNE), the reactor anomaly, and the results of the calibration of radiochemical experiments devoted to measuring solar-neutrino fluxes (for an overview, see [1]) favor the existence of sterile neutrinos. These anomalies can be described under the assumption that there exist one or two light sterile neutrinos of mass about a few eV units. The hypothesis of the existence of extra light neutrinos is compatible with present-day cosmological data obtained by measuring the anisotropy of cosmic microwave background radiation [2] and data on light-element abundances treated on the basis of the theory of primary nucleosynthesis [3]. Moreover, this hypothesis may remove [4, 5] currently existing contradictions between the observations of the Universe at small and large redshift values—namely, between the data of the Planck experiment, local measurements of the Hubble constant, and available statistics of large galaxy clusters. From the point of view of explaining dark matter, the range of sterile-neutrino masses between 1 and 10 keV is of greatest interest [6, 7].

By analogy with the three fermion generations in the Standard Model, we can expect that sterile right-handed neutrinos also form three generations. The discovery of one sterile-neutrino generation would strengthen the confidence in the existence of the remaining generations. If one of them forms dark matter, the mass-to-mixing angle ratio for this neutrino is substantially constrained by astrophysical observations, such as searches for a gamma line from sterile-neutrino decay in galactic halos [8] (for an overview,

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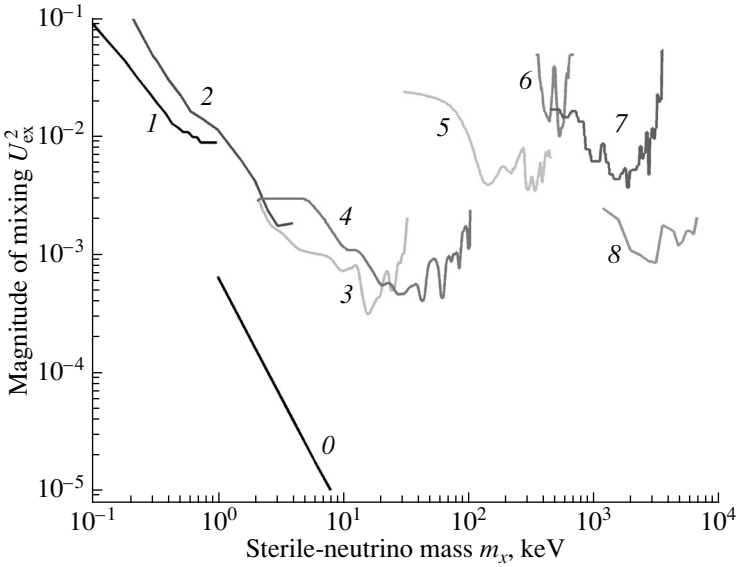

Fig. 1. Limits on the admixture of sterile neutrinos in the decays of the following nuclei: (*1*) ^{187}Re [13], (*2*) ^{3}H [14], (*3*) ^{63}Ni [12], (*4*) ^{35}S [15], (*5*) ^{64}Cu [16], (*6*) ^{37}Ar recoil nuclei [17], (*7*) ^{38m}K [18], and (*8*) ^{20}F [19]. The *0* symbol marks the line that represents the estimated sensitivity of the proposed experiment.

see [7]). These constraints do not apply to the remaining generations. In view of this, direct laboratory searches for sterile neutrinos are of particular interest both in the region of parameters that correspond to the dark-matter sector and beyond this region. The discovery of sterile neutrinos would provide a solution to a number of fundamental problems in particle physics (structure of the neutrino mass matrix, character of the extension of the Standard Model, and lepton-number nonconservation), astrophysics, and cosmology (dark matter).

2. SEARCHES FOR A MASSIVE-NEUTRINO ADMIXTURE IN BETA DECAYS OF NUCLEI

Dedicated searches for a massive-neutrino admixture in beta decays of nuclei were initiated by the pioneering article of Simpson [9], where it was claimed that an inflection point in the electron spectrum of tritium was observed in the region around 1.5 keV. The measurements reported there were performed with the aid of a Si(Li) semiconductor detector in which tritium was implanted. The observation in question served as a basis for concluding that a neutrino characterized by a mass of 17.1 keV and a mixing probability of 3% exists. After that, Simpson and Hime reported on two other experiments devoted to measuring the electron spectrum of tritium implanted in a high-purity germanium (HPGe) semiconductor detector [10] and the ^{35}S electron spectrum accumulated by an external Si(Li) semiconductor detector without source implantation in the detector [11]. In the quoted articles, the limit on the admixture in question was reduced to about 0.6%, but the presence of a heavy neutrino with a mass of about 17 keV was confirmed. Those results gave rise to a burst of experimental activities aimed at searches for such neutrinos in the decays of ^{14}C, ^{63}Ni, ^{177}Lu, and some other elements. Predominantly negative results that disproved Simpson's conclusions were obtained in a number of studies. The currently most stringent limit on the admixture of massive neutrinos was obtained with the aid of a magnetic spectrometer at the Zurich University in measuring the electron spectrum of ^{63}Ni. According to those measurements, the upper limit on the admixture of 17-keV neutrinos was estimated at 0.05% at a 95% confidence level (C.L.) [12].

Although the presence of an admixture of 17-keV neutrinos at a level of 0.6% was not confirmed by more recent independent experiments, searches for massive (sterile) neutrinos in beta decays of nuclei under laboratory conditions have been continued, albeit less vigorously. The present-day limits on the admixture in question are given in Fig. 1, where the upper limits on the mixing parameter U_{ex}^2 are shown for various values of the neutrino mass m_x. The simple case where a massive sterile neutrino ν_x is mixed with the electron flavor state ν_e of an active neutrino is considered. This figure was prepared by using the published results of recent laboratory experiments, where searches for an admixture of massive sterile neutrinos were performed on the basis of a detailed analysis of the decays of nuclei. The numbering of the lines on display corresponds to the experiments with (*1*) ^{187}Re nuclei [13], (*2*) ^{3}H nuclei [14], (*3*) ^{63}Ni

nuclei [12], (*4*) ^{35}S nuclei [15], (*5*) ^{64}Cu nuclei [16], (*6*) ^{37}Ar recoil nuclei [17], (*7*) ^{38m}K nuclei [18], and (*8*) ^{20}F nuclei [19]. In Fig. 1, we present the limits on the massive-neutrino admixture that were obtained for tritium with the aid of the magnetic spectrometers in Troitsk [20] and in Mainz [21]. Those experiments were aimed primarily at detecting the beta spectrum of tritium in the vicinity of the endpoint energy in order to measure precisely the ν_e mass; therefore, the limits on the admixture were obtained for m_x in the range of 1–100 eV. However, the plans of these groups for investigations in the near future (KATRIN project as a continuation of the experiment in Mainz) include measurements of the spectrum over a broader range that would make it possible to set limits for masses of about several keV units.

The same figure shows the upper limit expected for the above mixing parameter from the proposed experiment (the "*0*" symbol marks the respective line). This limit was obtained from a simulation of the experimental spectrum for a data sample of 10^{12} events. A more detailed account of the procedure underlying this estimation is given in Section 6 below.

3. SEARCHES FOR A MASSIVE-NEUTRINO ADMIXTURE IN TRITIUM BETA DECAY WITH THE AID OF A PROPORTIONAL COUNTER

From the point of view of the procedure of the experiment being discussed, the studies reported in [22, 23], where the beta spectrum of tritium was studied with the aid of a gaseous proportional chamber, are of greatest interest. Those studies were aimed at testing the hypothesis of 17-keV neutrinos; therefore, the segment of the spectrum between 0.8 and 3.5 keV was primarily studied for the presence of an inflection point. Despite this, some of their methodological features have a direct bearing on the proposed experiment. In particular, use was made of a gaseous Ar + 10%CH_4 mixture at a pressure of 0.5 atm. The maximum counting rate was 2000 Hz. About 10^9 events over the whole spectrum were accumulated within 500 h of measurements. It is of interest that the authors of [22, 23] did not indicate that they noticed a degradation of the gaseous mixture. Further, they mentioned the presence of a wall effect, which manifested itself primarily in the region of small energy depositions (up to about 3 keV). Those measurements led to setting an upper limit of 0.2% on the admixture of 17-keV neutrinos at a 90% C.L.

We would like to mention yet another early study that has a direct bearing on the proposed experiment. Here, we mean a measurement of the electron-(anti)neutrino mass by the shape of the tritium beta spectrum in the vicinity of the endpoint energy. A cylindrical proportional counter filled with a Xe + Ar + CH_4 gaseous mixture containing various concentrations of the components was used in [24, 25]. The activity of tritium in the counter did not exceed 30 000 events per minute (500 Hz). A degradation of the gaseous mixture was not reported. Also, the linearity of the scale and the stability of the energy resolution were highlighted. The tritium spectrum was studied in the region above 16 keV. The resulting limit on the neutrino mass was 1 keV. The aforementioned negligible contribution of the wall effect in the electron spectrum for energy depositions in excess of 5 keV was of importance from the methodological point of view. Although no numerical estimates of the wall-effect contribution were presented in [24, 25], an important conclusion can be drawn from those studies that a long-term accumulation of the spectrum at a high counting rate in the proportional counter is possible in principle since the wall-effect contribution will be insignificant.

4. NEW APPROACH TO MEASURING THE SPECTRUM OF ELECTRONS FROM TRITIUM IN A PROPORTIONAL COUNTER

As was indicated above, searches for a specific distortion that, owing to the hypothesized admixture of sterile neutrinos whose mass m_s ranges between 1 and 10 keV, arises in the energy spectrum of electrons from tritium decays are of interest from the experimental point of view. Below, we will show that this distortion extends over the whole spectrum; therefore, one can use detectors that have a relatively low energy resolution (about 10 to 15%). The electrons in question will be detected by a gaseous proportional counter. In the case of tritium, the use of a gaseous counter provides the advantage of combining a source and a target in a natural way; owing to a low threshold (down to 0.2 keV), this permits detecting almost the whole energy-deposition spectrum. In view of this advantage, gaseous detectors have been repeatedly chosen for measuring the spectrum of tritium [22, 24, 25]. The counter proposed for the experiment being discussed has a special structure. It is manufactured from quartz glass, and a thin carbon film serving as a cathode is deposited onto its inner surface [26, 27]. In contrast to traditional methods for manufacturing proportional counters, the fully fused quartz tube ensures here a radical improvement of the reliability and stability in operating the counter and provides the possibility of employing the counter over a broad temperature range extending up to 400°C.

A preliminary choice of counter structure and gaseous mixture is the following. The counter is a quartz tube 10 mm in diameter, its walls having a thickness of about 1 mm. A pyrolytic graphite layer

(cathode) of thickness about 1 μm is deposited onto the inner tube surface by means of the decomposition of isobutane, and a central tungsten filament 12 μm in diameter is used as an anode. In the case of filling with a $Xe + CH_4$ mixture at a pressure of about 1 atm, the pulsed current signal taken in the proportional mode directly from the anode filament has a duration of about 20 ns in a 10% base. The recording of such (with a step of about 1 ns) oscillograms of pulses will ensure a time resolution of about 3 to 5 ns and, as a consequence, will make it possible to deal with high counting rates.

The possibility of long-term measurements under conditions of a high rate of the appearance of useful events (counting rates of about 10^5 to 10^6 Hz) is a feature peculiar to the proposed experiment. The possibility of a long-term data accumulation at a high rate of counting of ^{3}H decays in the proportional counter is provided on one hand by the choice of operation conditions for the counter used (optimum gas mixture not prone to degradation, thermal stabilization of the counting equipment and high-voltage sources, and so on). On the other hand, the state-of-the-art level in the digital processing of signals permits ensuring the accumulation of vast data arrays in an online regime. Concurrently, there is no technical obstacle to recording, with a discrete step of about 1 ns, the shape of each current pulse coming from the counter and exceeding a preset threshold.

A similar regime of data accumulation was implemented at the Institute for Nuclear Research (Moscow), Russian Academy of Sciences, in studying the subbarrier fission of nuclei with a slowing-down lead (SDL) neutron spectrometer, as well as in studying the resonance structure of neutron cross sections with REPS and RADEX time-of-flight spectrometers. The experimental equipment and the measurement method used were described in detail elsewhere [28, 29]. As an example of employing quartz-tube counters, we would like to mention the SAGE experiment aimed at measuring the solar-neutrino flux [30]. In that experiment, counters of similar design have stably been operating for a long time.

The recording of detailed pulsed-signal oscillograms with a high time resolution permits substantially reducing the fraction of random superpositions of signals and accumulating a vast data sample in one detector within a reasonable time. For example, an energy-deposition spectrum formed by about 10^{12} counts in the energy range of 0.5–20 keV was accumulated in the counter within a month at a counting rate of 10^6 Hz. This data sample will make it possible to set an upper limit between about 10^{-3} and 10^{-5} on the sterile-neutrino admixture at a confidence level (C.L.) of three standard deviations (3σ) for m_s in the range of 1–8 keV. This approach to searches for the sterile-neutrino admixture in tritium decay has not yet been applied. Moreover, the accumulation of tritium-decay electron spectrum featuring about 10^{12} counts in a single detector over nearly the whole energy range would be a unique measurement not having analogs—the maximum number of events ever obtained for the tritium spectrum per measurement run with a proportional counter does not exceed 10^9 [22].

Methodologically, the approach described above was already implemented in [31]. However, the main objective of that study was to demonstrate the potential of the method of recording pulsed-signal oscillograms by addressing the example of measurements of ^{37}Ar activity in a proportional counter under conditions of a high counting rate. A discretization step of 10 ns proved to be sufficient for solving this problem. A pulsed current signal from the counter anode was formed by a preamplifier whose characteristic time was about 20 ns. A signal formed in this way had a pulse rise time of 50 to 70 ns and a decay time of up to 1 μs, the time resolution being 30 ns. Estimations of the quantities D and C characterizing the systematic distortion of the spectrum (see Section 8) were not performed in that study, but, on the basis of indirect data, these parameters can roughly be estimated at about 0.2% and about 1%, respectively. Within methodological investigations, similar measurements should be focused on estimating precisely these parameters.

5. MECHANISM RESPONSIBLE FOR THE DISTORTION OF THE SPECTRUM OF ELECTRONS FROM TRITIUM DECAY

Tritium ($Z = 1$, $A = 3$) decays according to the scheme

$$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e,$$

going over to the helium isotope ^{3}He, whose half-life is $t_{1/2} \simeq 12.3$ yr, the decay energy being $Q_\beta \simeq 18.6$ keV. The spin–parity of the nuclei involved in the respective reaction undergoes no change ($1/2^+ \to 1/2^+$), the decay belongs to the class of superallowed Fermi transitions, and the emitted electron has the s-wave form. For allowed transitions, the shape of the emitted-electron spectrum is given by the expression

$$\frac{dN}{dE} \sim F(Z,E)pE(E_0 - E)^2,$$

where p and E are the electron momentum and energy (in $m_e c^2$ units), respectively; E_0 is the endpoint energy; and $F(Z,E)$ is the Fermi energy, which takes into account the effect of the Coulomb interaction

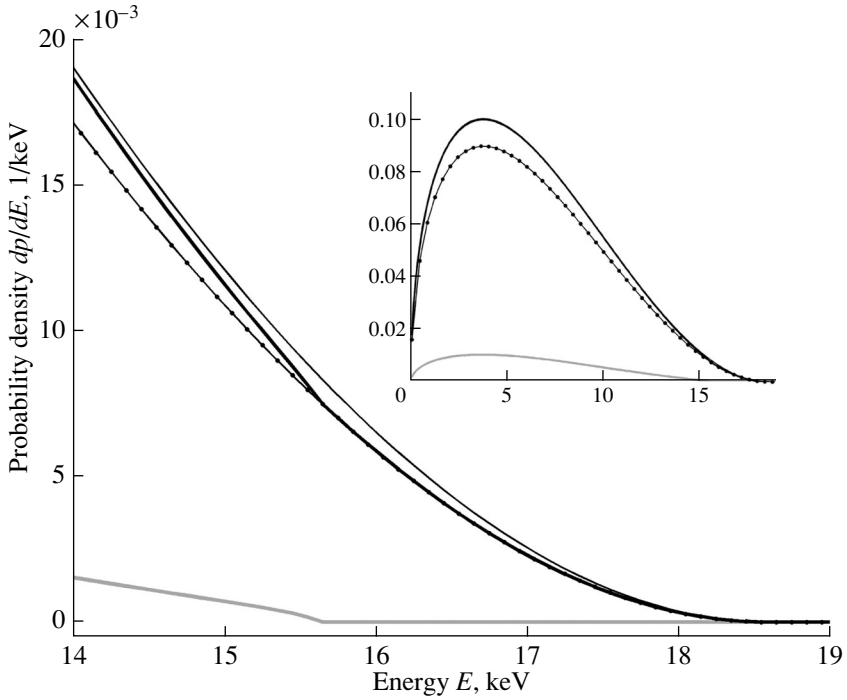

Fig. 2. Spectrum of electrons from tritium decay for a sterile-neutrino admixture in the range of 14–19 keV ($U^2 = 0.1$ and $m_s = 3$ keV): (thin line) pure spectrum S_l without an admixture of ν_s, (thin line connecting the points) same spectrum multiplied by the coefficient $1 - 0.1 = 0.9$, (thick gray line) pure admixture spectrum S_s multiplied by the coefficient 0.1, and (thick solid line) mixed spectrum $S = 0.9S_l + 0.1S_s$. The inset gives the electron spectrum over the whole energy range.

between the emitted electrons and the daughter nucleus. A nonzero neutrino mass m_ν leads to a shift of the endpoint energy of the spectrum, in which case we have

$$\frac{dN}{dE} \sim F(Z,E)pE(E_0 - E) \times [(E_0 - E)^2 - (m_\nu c^2)^2]^{1/2}.$$

In the presence of a massive sterile neutrino ν_s, the flavor state ν_e (and, accordingly, the state $\bar{\nu}_e$) can be represented in the form of a combination of light active (the respective index is i) and heavy sterile mass eigenstates as

$$|\nu_e\rangle = \sum_i U_{ei}|\nu_i\rangle + \sum_s U_{es}|\nu_s\rangle,$$

where U is the unitary lepton mixing matrix. Without loss of generality, this expression can be approximated by the mixing of only two mass eigenstates:

$$|\nu_e\rangle = U_{el}|\nu_l\rangle + U_{es}|\nu_s\rangle.$$

The effective mass of the light active neutrino ($m_l \lesssim 1$ eV) is negligible in relation to the mass of the sought sterile neutrino (about 1 keV). In that case, we can represent the spectrum of electrons from tritium decay as [32]

$$\frac{dN}{dE} = (1 - |U_{es}|^2)\frac{dN_l}{dE} + |U_{es}|^2\frac{dN_s}{dE},$$

where

$$\frac{dN_l}{dE} \sim F(Z=2,E)pE(E_0 - E)^2$$

is the pure spectrum featuring no sterile-neutrino admixture at zero active-neutrino mass and

$$\frac{dN_s}{dE} \sim F(Z=2,E)pE(E_0 - E) \times [(E_0 - E)^2 - (m_s c^2)^2]^{1/2}$$

is the pure admixture spectrum of sterile neutrinos. Hereafter, we disregard the spectrum of final states of the ^{3}He molecular ion.

In order to simplify our subsequent argument, we consider only the kinematical component of the electron-spectrum shape. For this, we get rid of the Fermi function in the analytic expressions used: we denote by $S_l \equiv \frac{dN_l}{dE}/F(Z=2,E)$ the pure spectrum featuring no sterile-neutrino admixture, by $S_s \equiv \frac{dN_s}{dE}/F(Z=2,E)$ the pure admixture spectrum of sterile neutrinos, and by $S \equiv \frac{dN}{dE}/F(Z=2,E)$ the mixed spectrum. For that case, the possible distortion of the emitted-electron spectrum is illustrated in Fig. 2.

Figure 2 shows the components of the mixed spectrum for the ad hoc option of $U^2 = 0.1$ and $m_s = 3$ keV. The thin line stands for the pure spectrum

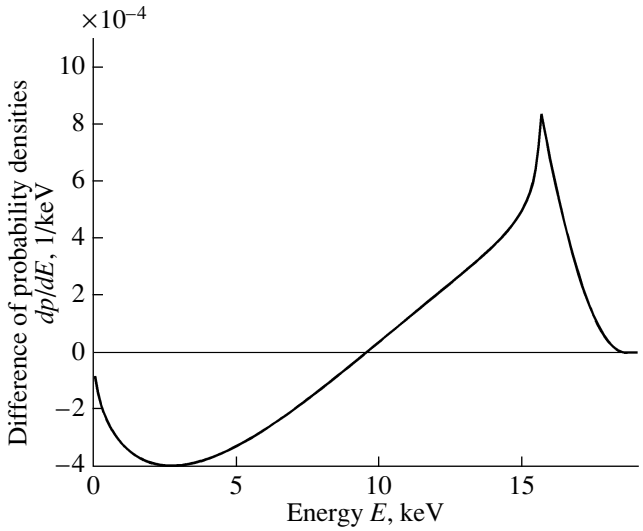

Fig. 3. Difference spectrum $S_l - S$ for an the hoc option of $U^2 = 0.1$ and $m_s = 3$ keV. The thin line indicates zero values.

S_l (featuring no admixture of ν_s), while the thin line going through points represents the same spectrum multiplied by the coefficient $1 - 0.1 = 0.9$. The thick gray line corresponds to the admixture spectrum S_s multiplied by the coefficient 0.1. The mixed spectrum $S = 0.9S_l + 0.1S_s$ is shown by the thick solid line. The spectra in question are normalized to unity in such a way that the meaning of the representation corresponds to the probability density dp for the emission of an electron that has an energy within a bin of width $dE = 1$ keV. It is noteworthy that, even for so strong a mixing, the spectrum S is virtually indistinguishable from the spectrum S_l; one can clearly see this in the inset, which gives the spectra over the whole energy range and where the thick solid line merges together with the thin line. The spectrum is distorted primarily within the segment offset from the endpoint energy E_0 by $m_s = 3$ keV—an inflection point on the thick solid line is observed in the region around 15.6 keV.

Figure 3 shows the difference $S_l - S$ of the emission probability densities, which provides a clear-cut characterization of special features of the inflection—a sharp peak at the inflection point against the background of the shape distortion over the whole spectrum.

6. SENSITIVITY OF THE METHOD: STATISTICAL UNCERTAINTY

The behavior described above makes it possible to employ integrals over the difference spectrum $S_l - S$ to estimate the magnitude of the distortion. The problem here is complicated by the fact that the measured spectrum S_{meas} is a superposition of the spectrum S_l not containing the sought effect and the effect proper, S_s, the reference spectrum S_l (featuring no admixture) not being measurable. However, this difficulty, which is typical of a broad range of physics problems, is relieved in the present case by the different character of the influence of the effect in question on different regions of the spectrum. In particular, it seems reasonable to employ an interval that corresponds to a negative difference (here, we have a vaster data sample and a less pronounced distortion of the shape of the spectrum in relation to the reference spectrum) to estimate parameters of the admixture-free spectrum. Thereby, the "negative" (in this sense) interval becomes a reference interval; accordingly, the "positive" interval will be a test interval. We will be interested in the difference of the counting rate based on fitting the admixture-free spectrum to the data in the reference interval and the counting rate actually measured in the test interval. The presumed effect receives contributions both from the distortion of the spectrum proper and from the bias of the estimate because of the effect of the sterile-neutrino admixture in the reference interval. In the case being considered, these contributions have the same sign; that is, they enhance the effect. As a matter of fact, the experiment is aimed at testing the hypothesis that there are no distortions in the measured spectrum.

Obviously, the boundary between the reference and test intervals will depend on the sterile-neutrino mass, and this will also permit in principle estimating this mass, provided that the effect is present. Within this approach, the full data-treatment procedure should therefore include maximizing the effect by choosing optimum boundaries between the reference and test intervals. Because of a preliminary character of our estimations, however, the reference and test intervals were taken to be fixed (in order to simplify

Values of $\mathcal{J}$ that are expressed in fractions of the normalized spectrum for various neutrino-mass values and several values of U_{es}^2

U_{es}^2	ν_s mass, keV				
	1	2	4	6	8
10^{-3}	2.7×10^{-6}	1.1×10^{-5}	4.4×10^{-5}	1.0×10^{-4}	1.7×10^{-4}
10^{-4}	2.7×10^{-7}	1.1×10^{-6}	4.4×10^{-6}	1.0×10^{-5}	1.7×10^{-5}
10^{-5}	2.7×10^{-8}	1.1×10^{-7}	4.4×10^{-7}	1.0×10^{-6}	1.7×10^{-6}

respective calculations) in evaluating the results of the simulation. Therefore, the resulting sensitivities are likely to be underestimated.

For a reference interval, we took the segment extending from 4 to 10 keV and containing about 60% of the whole spectrum. The lower boundary was taken to be rather high with the aim of suppressing the effect of uncertainty in the Fermi function $F(Z = 2, E)$. The choice of interval for fitting was quite arbitrary and has a preliminary character. Having obtained an estimate for the admixture-free spectrum S_l^{fit}, one can analyze the difference $S_l^{\text{fit}} - S_{\text{meas}}$ over the test interval between 10 and 18 keV. The choice of these boundaries for the intervals restricts the region of searches for sterile neutrinos to the mass range between 1 and 8 keV.

A high counting rate will make it possible to accumulate, within a reasonable time, an energy-deposition spectrum featuring about 10^{12} events. We will now estimate a limit that a data sample of this size will permit setting on the probability of the sterile-neutrino admixture. It was indicated above that, for a first approximation, the integral $\mathcal{J}$ of the difference spectrum over the test interval between 10 and 18 keV can be chosen as a measure of the distortion of the mixed spectrum: $\mathcal{J} = \int_{10}^{18}(S_l^{\text{fit}} - S_{\text{meas}})dE$. From the expression for dN/dE, it is clear that there is a linear relation between $\mathcal{J}$ and U^2. Numerical values of $\mathcal{J}$ that were expressed in terms of the fractions of the normalized spectrum are given in the table for various values of the sterile-neutrino mass.

If there is nosignal, one can find, at each value of the mass m_s, the smallest values of U^2 at which $\mathcal{J}$ still exceeds the statistical uncertainty at a 3σ level. For a data sample of 10^{12} events, this level for the neutrino mass in the range between 10 and 18 keV is $3\sqrt{0.35 \times 10^{12}} \sim 1.8 \times 10^6$ counts, which approximately corresponds to the 1.8×10^{-6} fraction of the spectrum. From the table, one can see that, at $m_s = 1$ keV, only for a mixing of about $U^2 = 10^{-3}$ does the value of $\mathcal{J}$ prove to be above this level. In other words, the upper limit on U^2 in the case of a data sample that comprises 10^{12} events (that is, the statistical sensitivity of the experiment) is 10^{-3} at $m_s = 1$ keV. At the mass value of $m_s = 8$ keV, the upper limit on the mixing is about 10^{-5}.

It is noteworthy that the above approach to estimating upper limits by using the data sample in question is quite rough and has a preliminary character, in just the same way as in the case of choosing the reference energy range for constructing a fit to the admixture-free spectrum. A detailed simulation will show whether it is possible to improve the sensitivity by analyzing the difference spectrum for the presence of special features—for example, a sharp peak (see Fig. 3).

In searches for a sterile-neutrino admixture and in the evaluation of the sensitivity, one can in general use a direct fit to the spectrum, adding two extra parameters—the heavy-neutrino mass and the squared sine of the mixing angle. However, this method features a strong dependence on the knowledge of the theoretical spectrum. The application of special statistical criteria constructed, for example, with the aid of the quasioptimum-weight method [33] may prove to be more convenient. It can be shown that this criteria are highly sensitive to the presence of the sterile-neutrino contribution to the spectrum but do not require precisely knowing the theoretical spectrum. Such criteria were developed, for example, in [34] with the aim of searches for a stepwise anomalous contribution to the spectrum of electrons from tritium beta decay in the Troitsk nu-mass experiment.

7. SYSTEMATIC DISTORTION OF THE AMPLITUDE SPECTRUM OF A PROPORTIONAL COUNTER

Systematic effects on the response of cylindric gaseous counters are well known. A dominant contribution to the counter response comes from the spread of measured pulse amplitudes, which is due, first of all, to the statistical spread of the number n of ion–electron pairs that arise upon primary ionization. It is common practice to characterize this spread by

the parameter R (energy resolution), which, by definition, is equal to the ratio of the counter-response FWHM for a given line to the energy of this line: $R = \mathrm{FWHM}/E$. The FWHM of a normal distribution and its variance σ are related by the equation $\mathrm{FWHM} = 2.34\sigma$; for large numbers, the variance is estimated as $\sqrt{n}$. In that case, $R = 2.34\sqrt{n}/n = 2.34/\sqrt{n}$; in the proportional mode characterized by a gas amplification of about 10^3, we accordingly have $R \propto 1/\sqrt{E}$, where E is the electron energy. For inert gases, the mean energy spent on creating an ion is about 30 eV. On average, about 200 primary-ionization electrons are generated by electrons of energy 5.9 keV that are knocked out in the counter working gas by an iron (^{55}Fe) x-ray source, which is frequently used for the purposes of calibration, the variance being $\sigma \sim \sqrt{200} \sim 14$ electrons. Thus, the counter response to electrons of energy 5.9 keV has a resolution not higher than $R_{5.9} = 2.34/14 \sim 16\%$; for an energy of 18 keV, the resolution is about 10%. Analytically, the resolution-induced smearing of the original spectrum can be represented in the form of the convolution

$$S_R(E) = \int_0^\infty S(x)G(\mu = 0, \sigma(x), x - E)dx,$$

where $S(E)$ is the original energy (E) distribution of electrons, $G(\mu, \sigma, E)$ is a normal (Gaussian) distribution characterized by an expectation value μ and an energy-dependent variance σ, and $S_R(E)$ is the distorted spectrum that arises upon the convolution.

A significant contribution to the systematic effect is due to the formation of the background pedestal in the spectrum because of the wall and edge effects, and we use the fraction D of events that involved in the pedestal as a quantitative measure of the effect. In the working mixture dominated by xenon, the cloud of primary ionization created by an electron of energy about 10 keV has a size of $l \sim 0.1$ mm at a pressure of 1 atm. In that case, the fraction of events in which the energy is degraded (wall effect) is $l/L \sim 1\%$ in a counter whose characteristic transverse size is $L \sim 10$ mm. A special counter design (conic shape of the endfaces) and a sufficient length (about 200 mm) make it possible to reduce the fraction of the boundary effect to about 0.1% (see, for example, [35]). The distortion of the original spectrum because of a rectangular pedestal whose fraction is D has the form

$$S_D(E) = (1 - D)S(E) + D\int_E^\infty \frac{S(x)dx}{x},$$

where $S_D(E)$ is the spectrum after distortion.

At a high counting rate, random coincidences (superpositions) of events come into play. The counting rate for coincident events, C, is in direct proportion to the product of the counting rate and pulse duration. Via recording the shape of each signal, it is possible to ensure a time resolution of about 3 to 5 ns, and the fraction C of indistinguishable superpositions that distort the final amplitude spectrum proves to be about 0.1% at a counting rate of about 10^6 Hz. The distortion of the original spectrum because of superpositions of pulses can be represented in the form

$$S_C(E) = (1 - C)S(E) + C\int_0^{\frac{1}{2}E} (S(x) + S(E - x))\,dx,$$

where the coefficient $1/2$ in front of E excludes double summation over the spectrum.

The distortion of the proportional-counter response because of the resolution, degrading, and indistinguishable superposition of events is illustrated in Fig. 4, where a model distribution of counter-pulse amplitudes is given. As specific examples, we present the response to 5- and 15-keV electrons that was subsequently distorted by using the respective factor. The notation in this figure is the following: the gray vertical lines stand for the original electron lines, each having an intensity of 10^{12} events; the points correspond to the smearing of the lines at a resolution of $R_{5.9} = 16\%$; the dashed line represents the distortion of the response because of the boundary and wall effects ($D = 1\%$); and the solid line corresponds to indistinguishable superpositions of events, the respective coefficient being $C = 0.1\%$. The amplitude spectrum of counter signals generated by tritium decays that was distorted by this response is presented in Fig. 5.

8. SENSITIVITY OF THE METHOD: SYSTEMATIC FACTORS

In addition to the statistical error, the systematic uncertainty also affects the sensitivity of experiment. By performing a simulation, one can estimate quantitatively the effect of the aforementioned systematic factors that contribute to the distortion of the spectrum.

The experimental spectrum S based on a specific data sample of N events for preset values of m_s and U^2 is formed at random by using the analytic expression for the probability density characterizing electron emission (see Section 5). After that, the spectrum obtained in this way is successively distorted by the resolution R, the pedestal D, and the superpositions

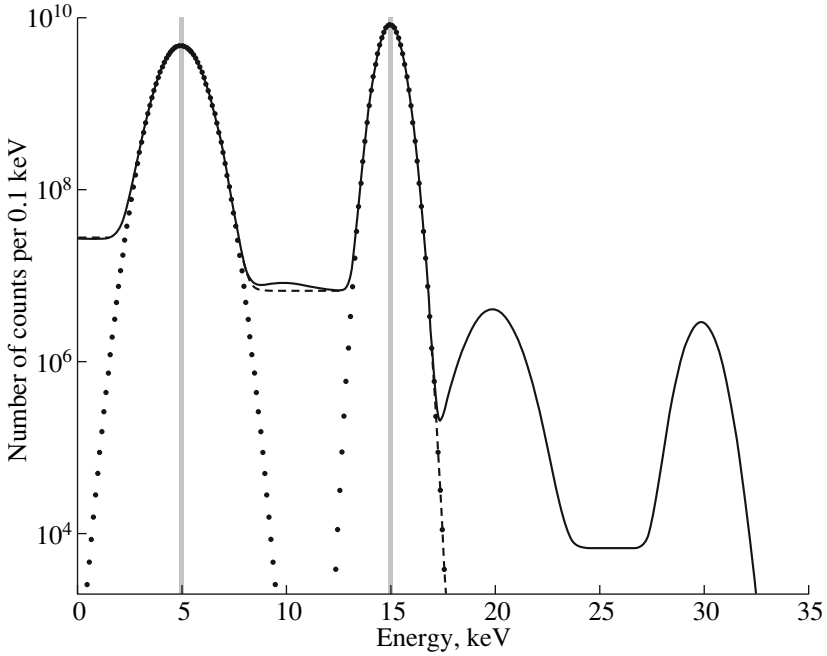

Fig. 4. Simulated counter response to 5- and 15-keV electrons (gray vertical lines, the number events being $N = 10^{12}$ for each of them) that was distorted by the resolution, degrading (wall and boundary effects), and indistinguishable superposition of events. We have $R_{5.9} = 16\%$ for the points, $D = 1\%$ for the dashed line, and $C = 0.1\%$ for the solid line.

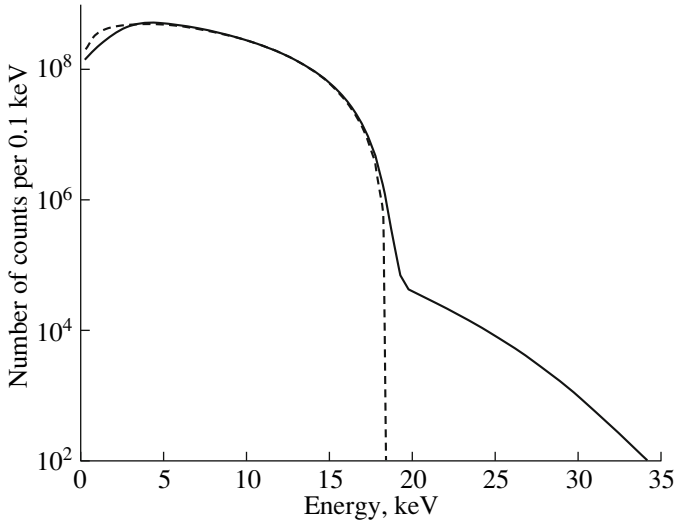

Fig. 5. Simulated spectrum of events in the counter in the presence of tritium for a data sample featuring $N = 10^{12}$ events: (dashed line) original spectrum of emitted electrons and (solid line) amplitude spectrum distorted by the resolution ($R_{5.9} = 16\%$), degrading ($D = 1\%$), and indistinguishable superposition of events ($C = 0.1\%$).

C. Therefore, the simulated spectrum S is a function of these parameters; that is,

$$S = S(m_s, U^2, N, R, D, C).$$

At the next step, S is fitted to the pure admixture-free spectrum S_l^{fit} distorted by respective systematic effects; that is,

$$S_l^{\text{fit}} = S_l^{\text{fit}}(N, R, D, C).$$

In doing this, one fixes the parameters D and C and employs N and R as adjustable parameters; that is, this fitting procedure yields, for N^{fit} and R^{fit}, specific values at which the admixture-free spectrum provides the closest approximation to the experimental spectrum. In applying the fitting procedure, use was made of the standard nonlinear-fit (nlinfit) function from the Matlab 7.0 package; the fitting was performed

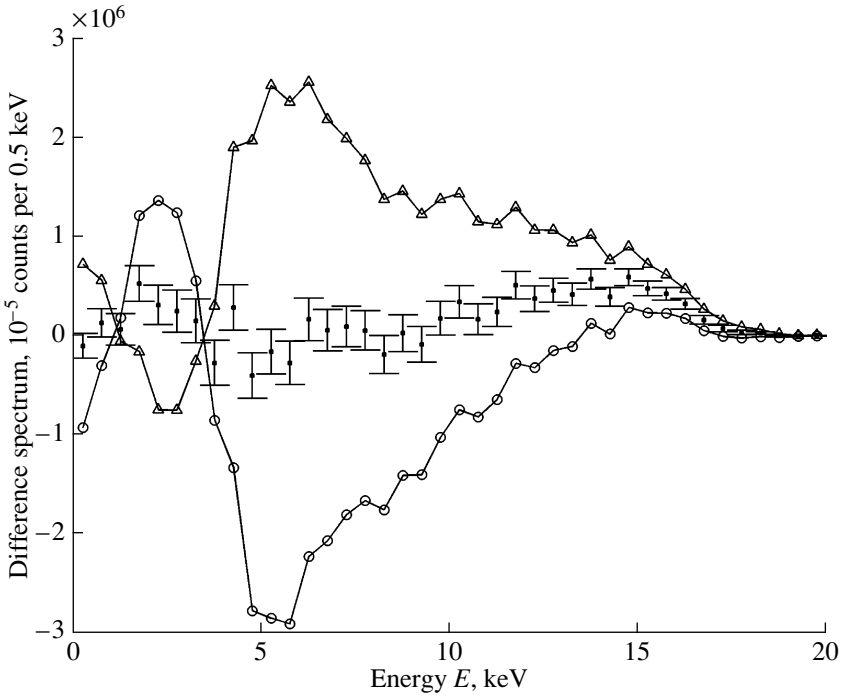

Fig. 6. Difference spectrum $S_l^{\text{fit}} - S$ for various pedestal values at the central values of $D_0 = 1\%$ and $C_0 = 0.1\%$ ($U^2 = 10^{-4}$, $m_s = 4$ keV, and $N = 10^{12}$). The fitted results are represented by closed circles with error bars for $D = D_0$ and $C = C_0$, open circles for $D = D_0 \cdot [1 - 0.01]$, and open triangles for $D = D_0 \cdot [1 + 0.01]$.

over the segment between 4 and 12 keV. Ultimately, an estimate of the signal $\mathcal{J}$ was obtained from the difference $S_l^{\text{fit}} - S$ as the integral of the difference spectrum over the segment between 12 and 18 keV.

From the point of view of estimating the systematic uncertainty, the spread of $\mathcal{J}$ values in response to variations in the fixed parameters D and C is of interest. This spread arises in the case where the simulated spectrum S is obtained for some pair of D and C values, while the fitting is performed for different values, $D \pm \delta D$ and $C \pm \delta C$. This situation simulates the inaccurate knowledge of D and C in a real experiment and, hence, provides an estimate of the systematic uncertainty.

As an illustration of the change in the simulated counter response upon the variation in the pedestal and superpositions, we present the results obtained by simulating difference spectra for the case of $U^2 = 10^{-4}$ and $m_s = 4$ keV; the simulated data sample comprises $N = 10^{12}$ events. Figure 6 gives the difference spectrum $S_l^{\text{fit}} - S$ for identical values of D and C at the central values of $D_0 = 10^{-2}$ and $C_0 = 10^{-3}$ (points with error bars). The same figure shows the differences in question for the case where one constructs a fit at different pedestal values: $D = D_0 \cdot [1 - 0.01]$ (open circles connected by a line) and $D = D_0 \cdot [1 + 0.01]$ (open triangles connected by a line). Similarly, Fig. 7 presents the same simulated difference spectrum along with fits at different superposition values: $C = C_0 \cdot [1 - 0.01]$ (circles connected by a line) and $C = C_0 \cdot [1 + 0.01]$ (triangles connected by a line). These figures clearly show that, even in the case of a small uncertainty (about 1%) in determining D and C, the signal may undergo substantial distortions, especially in the case of a pedestal.

The systematic uncertainty associated with an inaccurate knowledge of the pedestal and superpositions was estimated on the basis of a simulation in the absence of a sterile-neutrino signal in the mixed spectrum (case of $U^2 = 0$). The parameters D and C were varied within $\pm 1\%$ with respect to the central values of $D_0 = 10^{-2}$ and $C_0 = 10^{-3}$ for a statistical sample of $N = 10^{12}$ events. For each simulation, we fixed the amplitude of the spurious signal $\mathcal{J}$ generated by a systematic effect. The results are compiled in Fig. 8, where the signal amplitudes are given versus the change in D (thick gray line) and versus the change in C (thick solid line). Two thin lines indicate the boundary of the statistical uncertainty in zero signal at a 3σ C.L.; for a data sample of $N = 10^{12}$ events, this boundary corresponds to 1.8×10^6 counts (see Section 6).

On the basis of this figure, one can draw a conclusion that is of importance from the point of view of planning the experiment in question. The inaccuracy

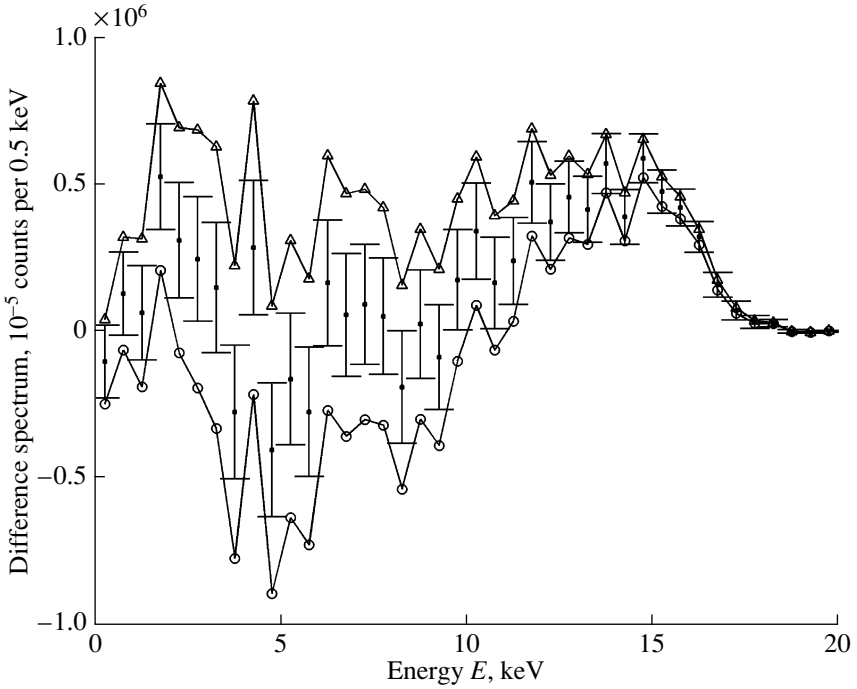

Fig. 7. Difference spectrum $S_l^{\text{fit}} - S$ for different superpositions at the central values of $D_0 = 1\%$ and $C_0 = 0.1\%$ ($U^2 = 10^{-4}$, $m_s = 4$ keV, and $N = 10^{12}$). The fitted results are represented by closed circles with error bars for $D = D_0$ and $C = C_0$, open circles for $C = C_0 \cdot [1 - 0.01]$, and open triangles for $C = C_0 \cdot [1 + 0.01]$.

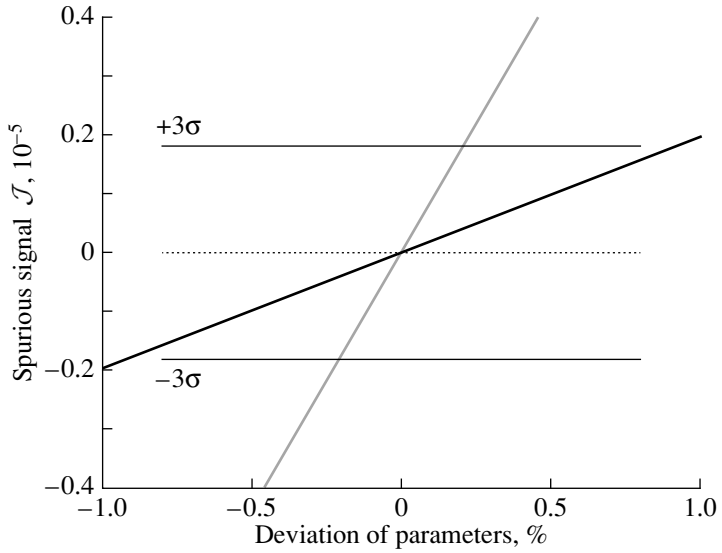

Fig. 8. Spurious signal $\mathcal{J}$ at zero mixing versus the change in the parameters D (gray thick line) and C (solid thick line), which characterize the pedestal and superpositions, respectively. The thin lines represent the boundaries of the statistical uncertainty in zero signal for $N = 10^{12}$ in the case of 1.8×10^6 counts, and the dotted line indicates the position of zero.

of available information about the pedestal (parameter D) has a much stronger effect on the ultimate result than the inaccuracy of knowledge of the superpositions (parameter C). In that case, calibrations that would ensure measurements of the pedestal to a precision of about 0.2% and superpositions to a precision of about 1% are necessary for the systematic uncertainty to be within the statistical uncertainty.

9. DISCUSSION

In order to minimize traditional factors contributing to systematic uncertainties, it is neces-

sary to choose appropriately the counter design and working-mixture parameters that would ensure the stability of counter operation under conditions of high counting rates. The implementation of the experiment under discussion would require manufacturing quartz-tube counters that feature carbon cathode sputtering and which have various diameters and lengths; for various mixtures of inert gases (Ar, Xe, and so on) with CH_4, CO_2, etc., with quenching additions, it is necessary to study an optimum composition and pressure that would ensure a fast operation and a long-term stability of the manufactured counters. On the basis of measurements, it would be possible to select the most appropriate counter and composition of the mixture for measurements with ^{3}H. Detailed calculations (numerical simulation) would be required for refining the contribution of various systematic-error sources. For calibration measurements, one can use inner (2.8-keV ^{37}Ar line) and outer (5.9-keV ^{55}Fe and 22-keV ^{109}Cd lines) sources. It is noteworthy that we are going to measure tritium spectra at the highest counting rate; therefore, a calibration of the counter filled with the working tritium medium is hardly possible. For this reason, an optimum way to calibrate the working-scale linearity and the counter energy resolution and stability is that of employing reference sources prior to and after measurements with a similar mixture.

Preliminary estimations show that, at average values of $D{\sim}10^{-2}$ and $C{\sim}10^{-3}$ precise to within $\pm 0.1\%$ and $\pm 1\%$, respectively, the contribution of the systematic uncertainty to the estimated limit on the sterile-neutrino admixture proves to be commensurate with the statistical-uncertainty contribution at a 3σ C.L. for the case where the total number of events in the spectrum is about 10^{12}. This precision in measuring the pedestal is reachable upon accumulating a data sample of not less than 10^8 events for calibration spectra, and this does not create any serious problem. It is noteworthy, however, that the simple rectangular-pedestal model underlying the estimation of the systematic effect can hardly provide a full description of the pedestal. Direct calibration measurements on the basis of a vast data sample would make it possible to refine our knowledge of the pedestal shape.

With the aim of obtaining a physically clear representation, the Fermi function was excluded from respective expressions in considering the mechanism of formation of the inflection point in the electron spectrum. A dominant contribution to the systematic distortion of the primary energy spectrum of electrons emitted in tritium decay comes from the Coulomb interaction of the outgoing electron with the daughter nucleus of ^{3}He. For the case of pointlike charges, the uncertainty in the calculated distortion, which is described by the Fermi function $F(Z, E)$, is restricted only by the inaccuracy of knowledge of the physics constants that appear in respective equations. At the present time, their inaccuracy does not exceed 10^{-8}. However, the simple Coulomb interaction in the real decay of tritium is in turn distorted because of a number of factors, including the screening of the daughter-nucleus field by the orbital electron, a finite size of the daughter nucleus, and electron exchange. In [36], Wilkinson considered in detail all possible processes that could correct the Fermi function and showed that, in the case of tritium, allowance for these factors leads to a multiplicative correction of about 10^{-3} or less, at least at a distance of 2 keV from the endpoint energy. The problem of accuracy in calculating $F(Z = 2, E)$ thereby reduces to that of the accuracy to which we know the corrections. In turn, the inclusion of the corrections is based on QED calculations—that is, a rather high accuracy may be expected. In any case, the uncertainty associated with the pedestal in the spectrum is much greater than the corrections to the Coulomb interaction. Thus, there is every reason to disregard the latter, at least in planning the experiment being discussed.

The contribution of decays of atoms absorbed by the counter walls will be a systematic-uncertainty factor peculiar to measurements with tritium. According to available data [22], this contribution is rather small in the case of employing hydrogen-free materials and the respective measurement procedure. Moreover, it can be monitored and taken correctly into account. In addition, some specialists in tritium interaction with materials (see, for example, [37]) report that hydrogen and tritium basically form a monomolecular layer on a flat surface of pyrolytic graphite deposited onto the inner surface of the quartz tube. This is one of the advantages of the application of a quartz-tube counter over a metallic cathode, in which case one first of all expects the diffusion of hydrogen from the surface of the respective metal to its interior.

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